# Refractive Index Profiling of Axially Symmetric Optical Fibers using Mach-Zehnder Interferometer and Model Matching Method 

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We propose a method for determining the refractive-index profile of axially symmetric optical fibers from transverse interferograms obtained by a Mach-Zehnder interferometer. The profile is obtained from the interferogram data by a modelfitting algorithm which uses the Abel transform of an approximate polynomial expression of the profile. This algorithm is simple to use, and does not require the Fourier transform. The proposed method is validated through computer simulations.

## 1. Introduction

Much research has been done on the measurement of the refractive index profile of optical fibers. In particular, the transverse interferometry method in which a ray is irradiated transversely onto the fiber is nondestructive and is highly practical [1-4].
However, the conventional technology put to practical use in the industry has a problem that complicated calculations such as Fourier transform are required. In this report, we propose a new measurement algorithm to obtain the refractive index profile by the model fitting method from the phase difference data obtained by the MachZehnder interferometer. The validity of the proposed method was confirmed by computer experiments.

## 2. Measurement principle

Configure a Mach-Zehnder interferometer as shown in Fig. 1. A laser (wavelength 546 nm ) is used as the light source, and irradiation is performed from the side of the optical fiber. The sample is immersed in index-matching fluid. The interferogram shown in Fig. 2 represents the optical path difference (OPD) due to the internal refractive index profile. Figure 3 shows the OPD obtained by the phase shifting method. The purpose of this research is to obtain the refractive index profile from this data.

### 2.1 Abel transformation

For a cylindrical object such as an optical fiber, where the refractive index is symmetrical, the OPD observed through the lateral direction is expressed as follows.

$$
\begin{equation*}
f(y)=2 \int_{y}^{R}\left[n(r) r / \sqrt{r^{2}-y^{2}}\right] d r \tag{1}
\end{equation*}
$$

where the periphery is a liquid for refractive index compensation, and the object refractive index $n(r)$ is represented by the difference from the periphery.
Here, the coordinate system is as shown in Fig. 4. This expression is called the Abel transform or Abel integral.

### 2.2 Abel inverse transformation

Obtaining $n(r)$ from $f(y)$ is the inverse Abel transform and is expressed by the following equation:

$$
\begin{equation*}
n(r)=-(1 / \pi) \int_{r}^{R}\left[(d f(y) / d y) / \sqrt{y^{2}-r^{2}}\right] d y \tag{2}
\end{equation*}
$$

The inverse transform of Eq. (2) is a definite integral that includes the derivative of the observed value function, is weak against noise, and is not easy in practice. Therefore, various methods have been proposed, including Fourier transform method [1-4] and Polynomial approximation method [5, 6].

These methods make the OPD data $f(y)$ differentiable by approximating it with a Fourier polynomial, etc. However, the Fourier transform has a problem that the calculation cost is high.

### 2.3 Proposed method

We have devised a model fitting method that assumes a refractive
index profile model for optical fibers. The observed values and the corresponding values predicted by the Abel transform of the refractive index profile model are least-squares fitted to obtain the unknown parameters in the model.

As the refractive index profile model, a polynomial model is adopted because its Abel transform is analytically possible. Here, we describe the case where the difference between the optical fiber refractive index and the immersion liquid refractive index is expressed as a function of the radius $r$ by the fourth-order polynomial eqation shown below:

$$
\begin{equation*}
\Delta n(r)=\Delta n_{0}\left[1-a(r / R)^{2}+b(r / R)^{4}\right] \tag{3}
\end{equation*}
$$

For this Abel transform we use the following indefinite integral formula:


Fig.1. Mach-Zehnder interferometer


Fig.2. Interferogram


Fig.3. Optical path difference profile


Fig.4. Coordinate system of Abel transform

$$
\begin{align*}
& \int\left[\left(1-a x^{2}+b x^{4}\right) x / \sqrt{x^{2}-y^{2}}\right] d x \\
& \quad=\sqrt{x^{2}-y^{2}}\left\{1-\frac{a\left(x^{2}+2 y^{2}\right)}{3}+\frac{b\left(3 x^{4}+4 x^{2} y^{2}+8 y^{4}\right)}{15}\right\} \tag{4}
\end{align*}
$$

Then the Abel transform of Eq.(3) becomes:

$$
\begin{equation*}
f(y)=2 \Delta n_{0} \sqrt{R^{2}-y^{2}}\left\{1-\frac{a\left(R^{2}+2 y^{2}\right)}{3 R^{2}}+\frac{b\left(3 R^{4}+4 R^{2} y^{2}+8 y^{4}\right)}{15 R^{4}}\right\} \tag{5}
\end{equation*}
$$

The parameters $\Delta n_{0}, a, b$ are obtained by the following least squares fitting of this equation to the observed values $f_{i}(i=1,2, \cdots, m)$.

$$
\begin{equation*}
\operatorname{SSE}\left(\Delta n_{0}, a, b\right)=\sum_{i=1}^{m}\left(f_{i}-f\left(y_{i}\right)\right)^{2} \tag{6}
\end{equation*}
$$

The refractive index profile can be calculated from the obtained parameters using Eq.(3)

## 3. Computer experiment

### 3.1 Experimental method

The OPD data was created under the following conditions.

- Central refractive index $n_{0}: 1.0$
- Refractive index model: 4th order polynomial (Eq.(3))
- Parameter a: 1.0
- Parameter b: 0.0
- Radius R: 1.0
- Observation interval: $1 / 20$ of the diameter
- Number of observation data (one side): 21

The obtained OPD data is shown in Fig.5. For the nonlinear leastsquares fitting, the Solver, which is an optimization tool of Microsoft Excel, was used. The initial values was set to $+10 \%$ of the true values. Note that this least-squares fitting problem can also be solved analytically by a variable transformation to a linear equations (see Appendix).

### 3.2 Experimental results

The results are shown in Table 1. The estimation error of the unknown parameter was less than 0.00001 . Figure 5 shows the matching results. Figure 6 shows the estimated refractive index profile obtained from the results. The estimation error (difference from the true value) was less than 0.00001 .

|  | True | Initial | Estimated | Error |
| :---: | :---: | :---: | :---: | :---: |
| $n_{0}$ | 1.0 | 1.1 | 1.0000 | 0.0000 |
| $a$ | 1.0 | 1.1 | 1.0000 | 0.0000 |
| $b$ | 0.0 | 0.0 | 0.0000 | 0.0000 |

## 4. Summary

We proposed a new inverse Abel transform algorithm for measuring the refractive index profile of an optical fiber using a Mach-Zehnder interferometer and Abel transform. Assuming an axially symmetric refractive index profile model, the observed OPD values and the corresponding values predicted by the Abel transform of the model are least-squares fitted to estimate the unknown parameters in the model. Polynomial approximation is effective as a refractive index profile model. For other functional models, their polynomial approximation can be used. The validity of the proposed method was confirmed by computer experiments. This method does not require complicated calculations such as Fourier transform, and the algorithm is simple.


Fig.5. Fitting results of OPD profile


Fig.6. Refractive index profiles

## Appendix: Analytical solution of least-squares fitting problem

Using the variable transformations of $X_{1}=\Delta n_{0}, X_{2}=$ $a \Delta n_{0}, \quad X_{3}=b \Delta n_{0}$ in Eq.(5), the model becomes linear as follows:

$$
\begin{equation*}
f(y)=A_{1} X_{1}+A_{2} X_{2}+A_{3} X_{3} \tag{A1}
\end{equation*}
$$

where

$$
\begin{align*}
& A_{1}=2 \sqrt{R^{2}-y^{2}}  \tag{A2}\\
& A_{2}=-2 \sqrt{R^{2}-y^{2}}\left(R^{2}+2 y^{2}\right) / 3 R^{2}  \tag{A3}\\
& A_{3}=2 \sqrt{R^{2}-y^{2}}\left(3 R^{4}+4 R^{2} y^{2}+8 y^{4}\right) / 15 R^{4} \tag{A4}
\end{align*}
$$

This linear least squares problem can be solved analytically as a simultaneous linear equation with three unknowns.

## References

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