

# Multi-wavelength single-shot interferometry without carrier fringe introduction

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## ABSTRACT

As a single-shot interferometric technique, spatial carrier interferometry has been thoroughly investigated, and it has been shown to have some problems, such as low spatial resolution. To overcome the problems, we propose a novel single-shot surface profiling technique that does not require carrier introduction. It is based on a model-fitting algorithm and estimates the model parameters and the heights of plural points simultaneously based on their multi-wavelength intensity data. The validity of the proposed method is demonstrated by computer simulations and actual experiments.

**Keywords:** Interferometry, single-shot, carrier fringe, surface profile, multi-wavelength

## 1. Introduction

The measurement accuracy of interferometric surface profiling is generally limited by environmental vibration. This is because the conventional methods use multiple images with different reference positions. To overcome this problem, researchers developed single-shot interferometry, which can measure the surface profile from a single image. A typical example is spatial carrier interferometry, in which carrier fringes are introduced by tilting the reference mirror. From a single interferogram, the phase distribution can be calculated by the Fourier-transform method <sup>1)</sup>, the spatial synchronous method <sup>2-3)</sup> or the Local Model Fitting method <sup>4)</sup>. However, spatial carrier interferometry has the disadvantages of low spatial resolution and a limited measurable slope angle. Also, this technique requires a long coherence length illumination source and optics with long depth-of-focus imaging.

This paper proposes a new algorithm for single-shot interferometry that does not require spatial fringe introduction. This technique is called the Global Model Fitting (GMF) method, and its most significant feature is that there is no loss in spatial resolution. We proved the validity of the GMF method by computer simulations and actual experiments.

## 2. Principle

In this chapter, we introduce two algorithms: the GMF method and the ACOS method. The GMF method is a novel technique that estimates the surface height of each pixel from its color information. Because of the high computational cost, this method is applied to a limited number of pixels in practical use, and the heights of the other pixels are calculated fast by the ACOS method using the information obtained by the GMF method.

### 2.1 GMF Algorithm

#### 2.1.1 Relation between color and height in three-wavelength interferometry

Let's consider an optical configuration for the three-wavelength single-shot interferometry shown in Fig. 1. This is almost the same configuration as that reported by Kitagawa<sup>5)</sup>. Fig. 2 shows the relation between color and surface height in three-wavelength interferometry. The upper is an experimentally obtained interferometric color image when the surface is tilted as shown in the lower graph. The height in the figure is roughly estimated by the RGB intensity data and the fact that the object is a flat surface. The color changes due to the cyclic variation of BGR signals according to the optical path difference (OPD) between the target and reference surface. Please note that the period of the cyclic signal depends on the illumination wavelength.

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This means that we can estimate the surface height of each pixel from its color information without carrier fringe introduction, even when the target surface is not flat. This interference color phenomenon is the same as that seen in soap bubbles. Its typical application is transparent film thickness measurement, where the white-light interference color is analyzed by spectroscopy. Another application is wavelength scanning interferometry for absolute distance measurement or surface profilometry, where the OPD is obtained by counting fringe cycles during the wavelength scanning<sup>6)</sup>.

The principle drawback of white-light systems is that they are just too slow. The solution to this problem is to settle on a small number of discrete wavelengths and measure the corresponding intensities directly, rather than scanning through the spectrum. One example already proposed is the head slider flying height measurement system described by Kubo et al.<sup>7)</sup>, in which a color camera captures the interference image and the flying height at each pixel is estimated from its hue. The relation between hue and the flying height must be obtained in advance.

Similarly it would be possible to estimate the surface profile from the color image based on calibration data obtained in advance using a test surface with known heights. However, there would be two problems with such a scheme. One is determining how to estimate the height from the color, and the other is that the relationship between height and color can change easily depending on the optics and surface conditions.

To overcome these problems, we propose a new technique, described in the next section.

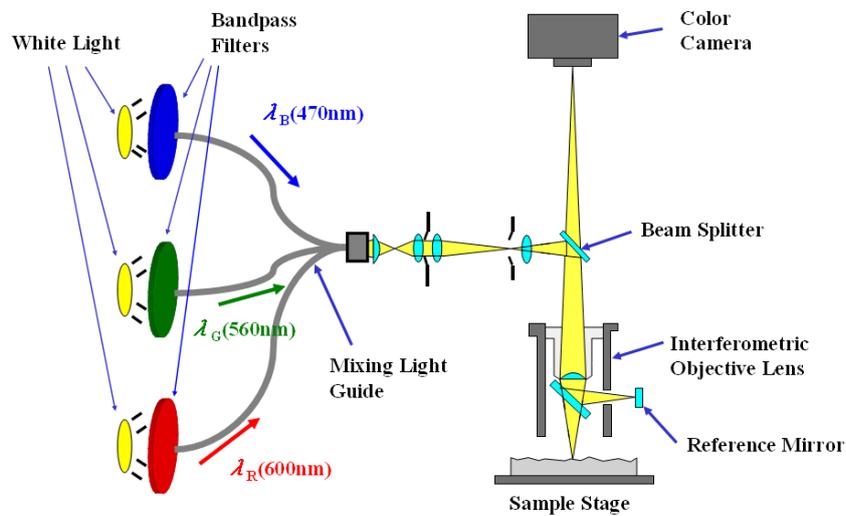


Figure 1. Optics of three-wavelength single-shot interferometry.

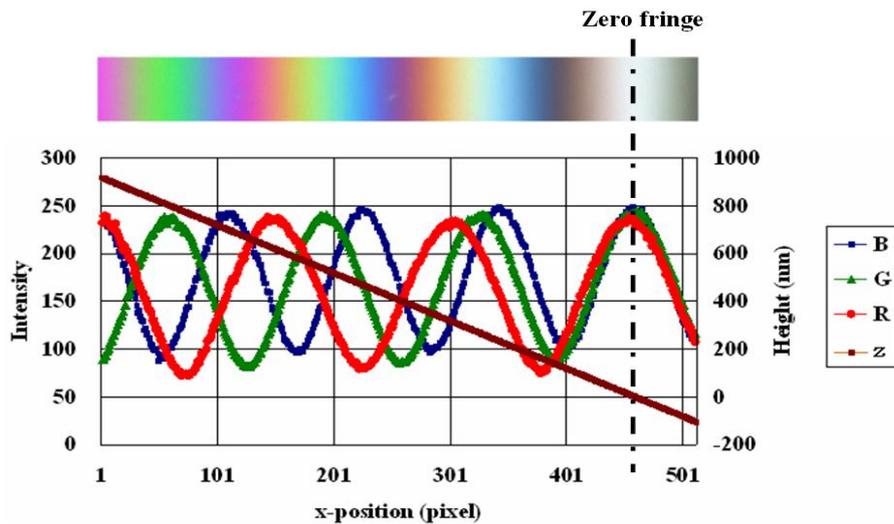


Figure 2. Relation between color and surface height in three-wavelength interferometry.

### 2.1.2 The GMF algorithm

There are three wavelengths in Fig. 1 and Fig. 2, but in the following equations we assume a more generalized expression of  $m$  wavelengths.

When we capture interferometric images of  $m$  wavelengths, the observed intensity is given by the following model:

$$g(i, j) = a(j)[1 + b(j) \cos\{\phi(i, j)\}], \quad (1)$$

where  $g(i, j)$  is the intensity at point  $i$  ( $i = 1, 2, \dots, n$ ) and wavelength  $j$  ( $j = 1, 2, \dots, m$ ),  $a(j)$  and  $b(j)$  are the average value and the modulation of the waveform, respectively, and  $\phi(i, j)$  is the phase given by

$$\phi(i, j) = 4\pi z(i) / \lambda_j, \quad (2)$$

where  $z(i)$  is the height relative to the zero OPD, and  $\lambda_j$  is the wavelength of the wavelength number  $j$ .

Equation (1) can be expressed by

$$g(i, j) = a(j)[1 + b(j) \cos\{4\pi z(i) / \lambda_j\}]. \quad (3)$$

This model is derived under the assumption that the waveform parameters  $a(j)$  and  $b(j)$  are constant in the field of view (FOV) and dependent only on the wavelength. This assumption will be almost always valid when the target surface is homogeneous.

The unknown parameters  $a(j)$ ,  $b(j)$  and  $z(i)$  can be estimated using the following least-square fitting equation:

$$J[a(j), b(j), z(i)] = \sum_{i=1}^n \sum_{j=1}^m [g(i, j) - g_{ij}]^2, \quad (4)$$

where  $g(i, j)$  is the model intensity defined by Equation (3) and  $g_{ij}$  is the observed intensity.

This non-linear least-square problem can be solved by any numerical methods. In this paper, we used the Solver program in MS Excel for the computer simulation. For the actual experiments, we used our own program based on the Davidon-Fletcher-Powell algorithm.

Please note that there is no loss in spatial resolution, that is, the height of each point can be estimated independently without any information about its neighboring points.

### 2.1.3 Necessary conditions

Let us consider the necessary conditions to obtain the unknown parameters  $a(j)$ ,  $b(j)$  and  $z(i)$ . When the number of wavelengths is  $m$  and the number of points is  $n$ , the total number of unknown parameters is  $2m+n$ . Since  $m$  values are observed at one point, the necessary condition for the solution is  $mn \geq 2m+n$ . Then, the number of points must be

$$n \geq 2m/(m-1) \quad (5)$$

This means  $n \geq 4$  in the case of  $m=2$ , and  $n \geq 3$  in the case of  $m=3$ . When  $n = 2m/(m-1)$ , then the problem becomes a  $(2m+n)$ -order non-linear simultaneous equation, and when  $n > 2m/(m-1)$ , then the problem becomes a  $(2m+n)$ -order non-linear least-square problem.

Figure 3 illustrates the principle of this algorithm in the case of three wavelengths and  $n$ -points. From  $3n$  observed intensities in the yellow cells,  $(n+6)$  unknown parameters in the blue cells, i.e.,  $n$ -point heights and six waveform parameters, are estimated.

Please note that we can select any arbitrary points for fitting. To obtain good estimation, it is advisable to select the points so that their height distribution becomes wide. When the target surface is fairly flat, it should be slightly tilted, as shown later in Fig. 13.

There is one important detail we must take care of. As shown in Fig. 2, the color is symmetrical along the zero OPD. Therefore, it is essential to make the OPD only positive or negative everywhere in the FOV by adjusting the  $z$ -axis.

		Observed data			To be estimated				
		$z$	B	G	R	$z$	B	G	R
Position	1								
	2								
	...								
	$n$								
		Intensities			Heights				
Model	$a$								
	$b$								
					Parameters				

Figure 3. Principle of the GMF method (in the case of 3 wavelengths and  $n$ -point fitting)

### 2.1.4 Initial estimates

To find solutions to the non-linear least-square problem described in the previous section, we use an iterative technique, which

requires initial estimates that will allow us to search for the minimum. Since the model function of Equation (3) contains a cosine function, the error function has a lot of local minimum. Therefore, it is essential to make good initial estimates.

In this paper,  $a(j)$  is set to be the average of the observed values and the modulation,  $b(j)$ , is set to be the range of observed values divided by  $2a(j)$ . The height,  $z(i)$ , is a rough estimate which is given usually by a priori knowledge of the target surface.

## 2.2 ACOS Algorithm

The computational cost of the non-linear least-square problem is very high, so the method becomes impractical when the number of points is large. Therefore, we use the GMF algorithm as the first step with a small number of points, for example, under one hundred, and then the heights of the other points are calculated as the second step by the following method, named the ‘‘arccosine(ACOS)’’ method, which uses the estimated waveform parameters from the first step.

### 2.2.1 Phase estimation

When the waveform parameters are given, the phase is obtained from the observed intensity by the following equation derived from Equation (1):

$$\phi(i, j) = \cos^{-1}[\{g_{ij} / a(j) - 1\} / b(j)], \quad (6)$$

where  $\cos^{-1}$  is the arccosine function and its value range is  $[0, \pi]$ . When the argument of the function is not within  $[-1, 1]$ , the function is undefined. In this case, the argument is approximated as  $-1$  or  $1$ .

### 2.2.2 Phase unwrapping

From the phase, the height,  $z(i, j)$ , is obtained by

$$z(i, j) = [\pm\phi(i, j) / 2\pi + N(i, j)](\lambda_j / 2), \quad (7)$$

where  $N(i, j)$  is the fringe order (integer), which is estimated by the coincidence method<sup>5)</sup>. The principle of this method is the same as the so-called exact fractions method for gauge block length measurement by multi-wavelength interferometry.

Fig. 4 shows an experimental example of using this method. For each wavelength, the heights with different orders are plotted. The unknown orders are determined so that the three candidate heights match best. In this case, the height is estimated as 300 nm. It should be noted that the phase is obtained by the arccosine function, not by the arc tangent function. Therefore, there are two candidate heights for each fringe order, as shown in Fig. 4.

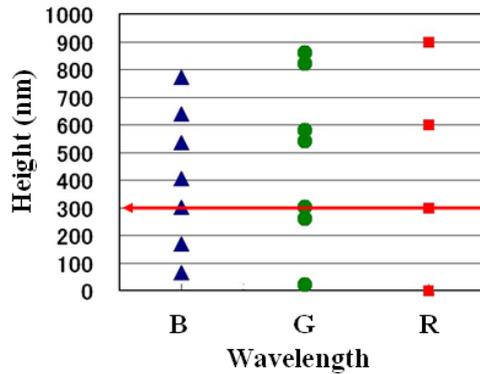


Figure 4. Phase unwrapping by the coincidence method.

## 2.3 Flowchart

The flowchart of the proposed method is shown in Fig. 5. Step 1 is the GMF method, and the heights and the waveform parameters are obtained by a least-square fitting. In Step 2, the obtained parameters are used to estimate the heights of other points. With the parameters, the height of any point can be estimated from its intensity data by the much faster and simpler calculation algorithm of the ACOS method. The parameters can also be applicable to other images captured under the same optical conditions. For that reason the collection of parameters can be called a recipe, as seen in the flowchart.

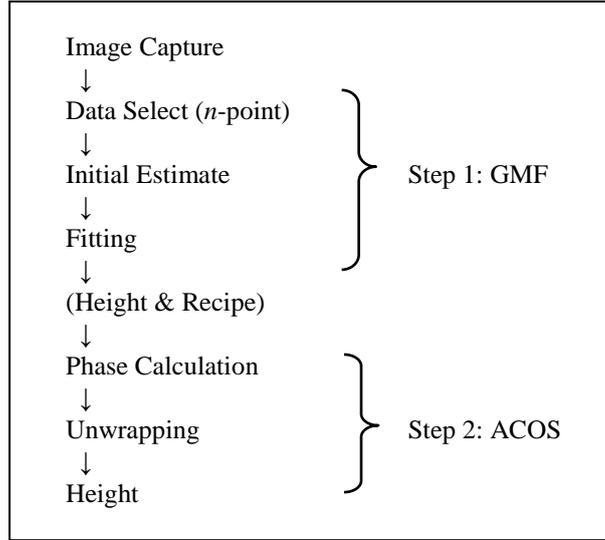


Figure 5. Flowchart of the proposed method.

### 3. Computer Simulations

#### 3.1 Test method

A three-wavelength interferometric color image was synthesized with the following conditions: (a) image size = 50×50 pixels; (b) pixel size = 1  $\mu\text{m}$ ; (c) wavelength = 470, 560, 600 nm; (d) target surface = sphere with 1-mm radius, with a small square protrusion of height 50 nm and size 4×4 pixels; (e) waveform parameters of  $a = 100$  and  $b = 1$ . The target surface is shown in Fig. 6(a), and the synthesized image is shown in Fig. 6(b). All the computations were done in MS Excel with a Windows PC. The non-linear least-square fitting in the GMF method was executed by the Solver program in MS Excel.

We performed two experiments. The first one used 3 points for fitting, and the second one used 50 points. The coordinates of the sampled points were (5, 25), (15, 25), (25, 25) in the first test and (1, 25), (2, 25), ..., (50, 25) in the second test.

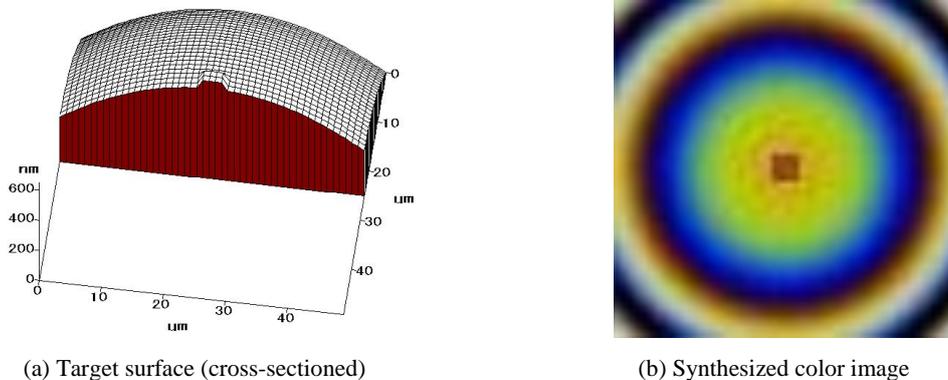


Figure 6. Target surface and its color fringe image used for computer experiments.

#### 3.2 Test results

##### 3.2.1 Three-point fitting

The RGB images are shown in Fig. 7 with three points used for fitting. The intensity values are shown in Fig. 8(a). The initial estimates of the heights are set at 95% of the true ones. The data for fitting and the results are shown in Table 1 and Fig. 8(b), respectively. The parameters and heights are estimated correctly.

Next, we estimated the height profile of 50 points along the  $y = 25$ . The intensities and calculated phases are shown in Fig. 9(a)(b). The heights are estimated almost correctly, as shown in Fig. 10.

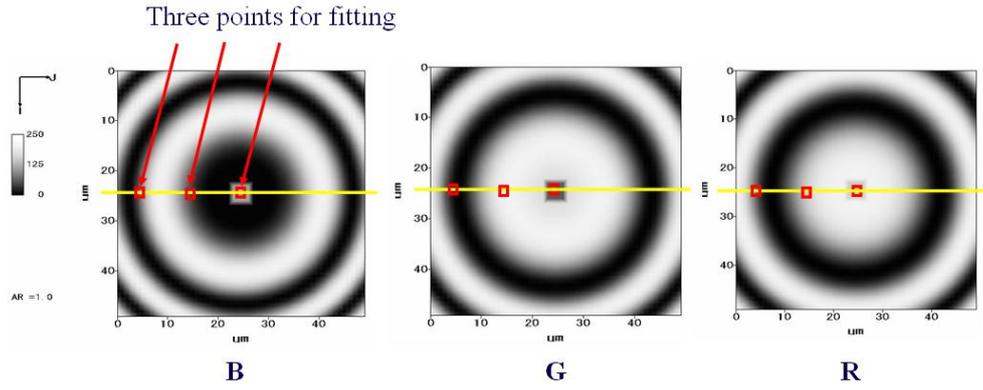


Figure 7. Fringe images of three colors and three points used for fitting.

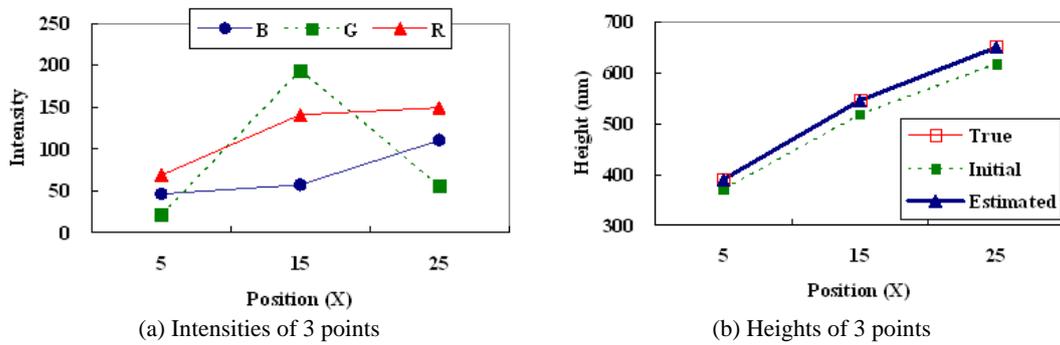


Figure 8. Observed intensities and the estimated height by the 3-point fitting.

Table 1. Estimated results by the 3-point fitting.

x-position	z(true)	Observed intensity			Estimated				
		B	G	R	z	Parameter	B	G	R
5	390	47	22	69	390	<i>a</i>	100.05	99.97	99.95
15	545	57	195	141	545	<i>b</i>	1.00	1.00	1.00
25	650	110	56	150	650				

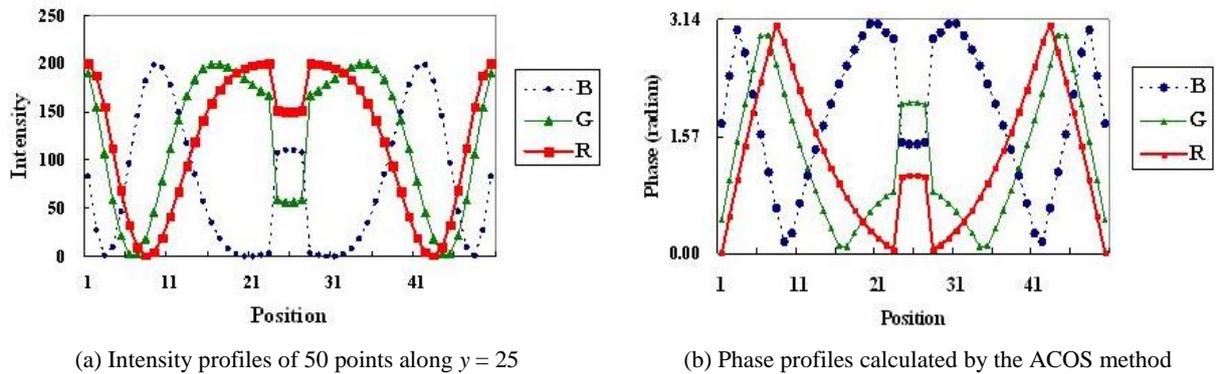


Figure 9. Observed intensities and phase profiles in the second step.

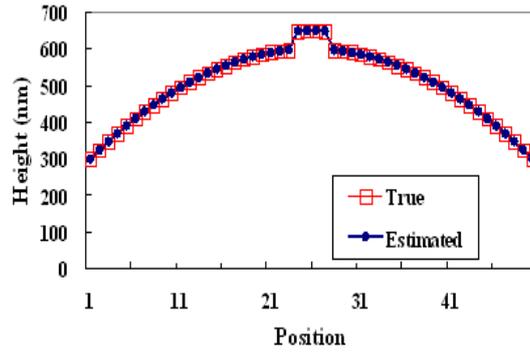


Figure 10. Estimated heights of 50 points by the ACOS method with the recipe obtained by the GMF 3-pt fitting.

### 3.2.2 50-point fitting

We estimated the height profile of 50 points along the  $y = 25$  using 50 points for the GMF fitting. The intensities are shown in Fig. 9(a). The initial estimates are set to be the true heights minus 50 nm. The heights are estimated correctly, as shown in Fig. 11. Please note that the small protrusion of  $4 \times 4$  pixel size is measured correctly without any loss in spatial resolution. This would be impossible by the conventional spatial carrier method.

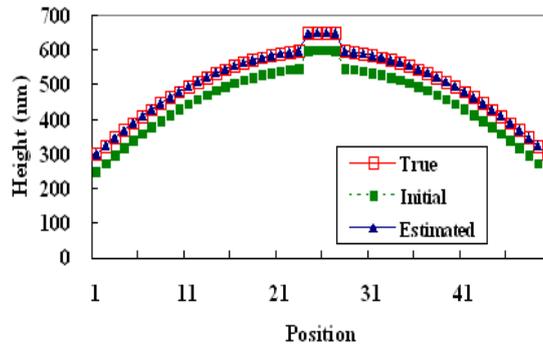


Figure 11. Estimated heights of 50 points by the GMF 50-pt fitting.

## 4. Actual Experiments

### 4.1 Test method

We installed the GMF algorithm in the optical surface profiler SP-500 by Toray Engineering Co., Ltd. (Fig. 12)<sup>8)</sup>, some of which was modified for three-wavelength single-shot interferometry as follows. The illumination unit was three halogen lamps, three narrow-band filters with 10-nm bandwidth and a mixing light guide with three branches, as shown in Fig. 1. The central wavelengths of the filters were 470 nm, 560 nm and 600 nm. The camera was a three-CCD color camera with  $1360 \times 1024$  pixels. The captured image data was stored in a PC memory as an integer value from 0 to 255. The color crosstalk was compensated by the crosstalk compensation algorithm reported in Ref. [5].

We measured the surface profile of a 50-nm step height. The surface was slightly tilted so that the interference image had different colors in the field of view. Fig. 13 is a central portion of  $512 \times 480$  pixels of the captured image. For simplicity we calculated this limited area.

The non-linear least-square equation in the GMF algorithm was solved using the Davidon-Fletcher-Powell method<sup>9)</sup>. The data for fitting were 20 points of  $x = 25, 50, 75, \dots, 500$  and  $y = 240$ . In the second step, we calculated the height of the whole area of  $512 \times 480$  pixels by using the parameters obtained in the first step.



Figure 12. Surface profiler used for experiments.

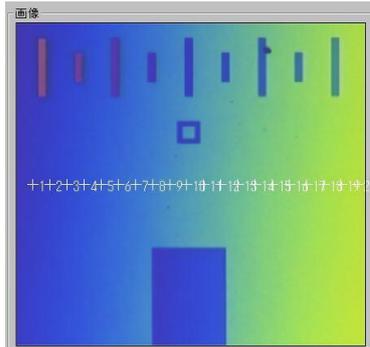


Figure 13. Captured color fringe image of 50-nm step height (512×480 pixels) and 20 sampled data points (shown in white numbers).

#### 4.2 Test results and discussion

The 20-point intensity data are shown in Fig. 14. From these data, the heights of 20 points were estimated by the GMS method as shown in Fig. 15. The results agree with the expected flat surface. Then the heights of the whole area were measured by the ACOS method, and the results are shown in Fig. 16 and Fig. 17. Notice that the bottom areas of small slits are measured without smoothing out sharp edges. This shows the significant feature of the proposed technique that there is no loss in spatial resolution. It has been impossible by the conventional spatial carrier interferometry. Another advantage is its measurement speed. The total calculation time for 512×480 pixels was about 0.5s including 0.1s of the GMF fitting using a C language program running on a Windows PC with Intel Pentium M (1.6 GHz). Because the calculation of the ACOS method is pixel-independent, the speed would be very much improved by a parallel processing technique.

The tests results have proved the validity of the proposed method. However, we can notice some errors in the results. To solve this problem and to extend the measurement range are still be investigated.

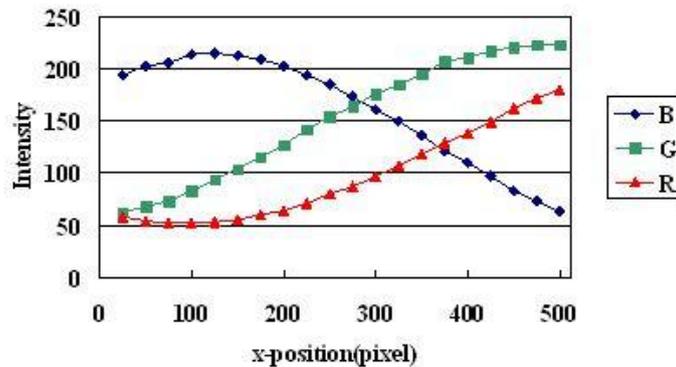


Figure 14. RGB intensity data of 20 points used for fitting.

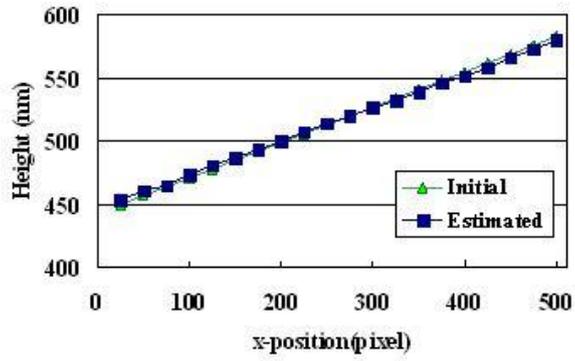


Figure 15. Estimated heights of 20 points obtained by the GMF method.

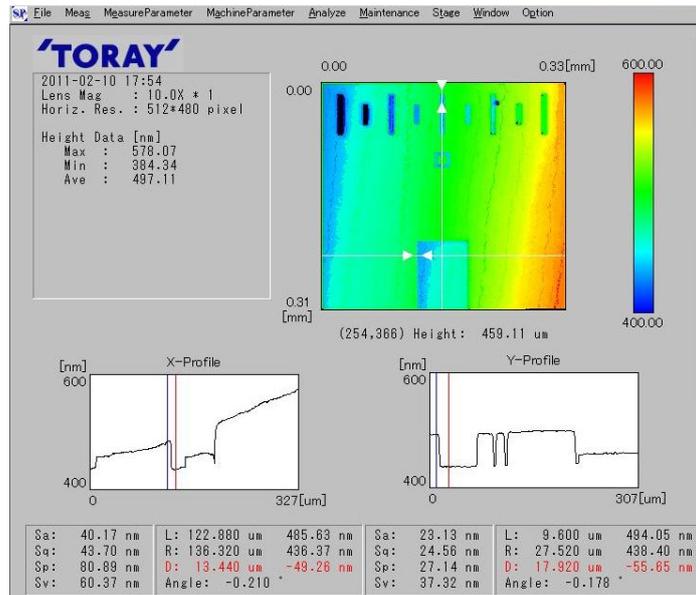


Figure 16. Estimated heights at 512×480 pixels by the ACOS method. The upper panel is a 3-dimensional map, and the lower graphs show x- and y-profiles.

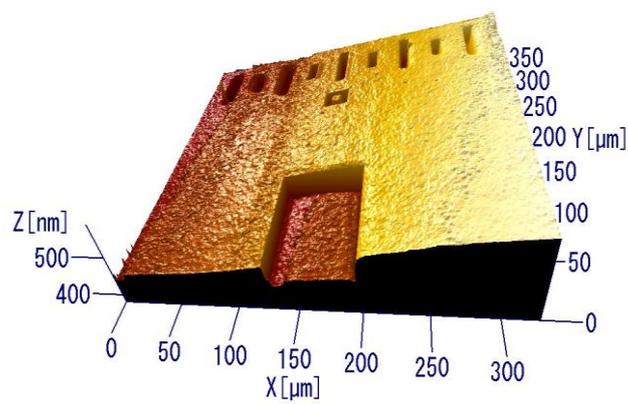


Figure 17. Estimated heights of the 50-nm step height in 3D view.

## 5. Conclusion

We have proposed a new single-shot interferometric surface profiling technique, the Global Model Fitting (GMF) method, which enables us to measure a surface profile without the need for carrier fringes. It is based on a model-fitting algorithm and estimates the model parameters and the heights of plural points simultaneously from their multi-wavelength intensity data. When the number of the wavelengths is three, the minimum number of data points is three, and a total of nine unknown parameters including the heights of three points are estimated by least-square fitting. Once the waveform parameters are estimated by this technique, they can be used for height estimation of the points other than the fitted points, which can be executed by a much simpler and faster algorithm using the arccosine function.

The most significant feature of this technique is that there is no loss in spatial resolution, that is, the height of each point can be estimated independently without having to know any information about its neighboring points. The validity of the proposed method has been proved by computer simulations and actual experiments.

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