# High-Precision Absolute Distance Measurement of Surface Topography using Wavelength Scanning Interferometry

# Katsuichi KITAGAWA<sup>1</sup> and Jun MIZOJIRI<sup>2</sup>

<sup>1</sup>Independent Consultant, Shiga, Japan <sup>2</sup>Mizojiri Optical Co., Ltd., Tokyo, Japan

Abstract: Wavelength scanning interferometry is known as a method to measure the surface topography of an object with high accuracy. However, it is difficult to obtain high-precision absolute measurements with conventional techniques. The authors have developed a new algorithm for measuring surface topography using a Fizeau interferometer and wavelength scanning interferometry, which measures absolute distances on the order of 10 mm with the accuracy on the order of nm. The validity of the proposed method was confirmed by computer experiments.

# 1. Introduction

Wavelength scanning interferometry is known as a method to measure the surface roughness of an object in three dimensions with high accuracy. In wavelength scanning interferometry, a wavelength tunable laser is used as the light source, and the wavelength is scanned linearly in time. Using the fact that the frequency of the light intensity signal is proportional to the optical distance, the distance to the object can be determined. In addition, even if there are multiple interfaces, such as in the case of a transparent plate, they can be separated and analyzed.

The authors have already reported wavelength scanning (or wavelength shifting) interferometry algorithms for transparent objects [1, 2, 3]. All of them are based on the model fitting method and assume a smooth surface without large steps.

On the other hand, there is a need in the industry for surface profiling that includes large steps beyond 1/2 wavelength. White light interferometry is a conventional technique, but it has the disadvantages of low accuracy and moving parts.

The measurement algorithms for opaque objects using wavelength scanning interferometry can be classified as follows.

1) Frequency detection method: For example, the zero-crossing method [4] and the Fourier transform method [5, 6].

2) Phase gradient detection method: The distance is calculated from the gradient of the phase change with wavelength. There are several methods to calculate the phase, such as the zero-crossing method [4], the window function method (similar to the phase shift method, the phase is calculated from the neighboring intensity values) [7], the Fourier transform-inverse transform method [8], and the sinusoidal function fitting method [9].

However, with these conventional techniques, it is very difficult to achieve a measurement accuracy of several nm in a measurement range of several mm. In this paper, we propose a new measurement algorithm that solves these problems.

## 2. Measurement Principle

### 2.1 Wavelength scanning model

A Fizeau interference optical system is constructed as shown in Fig. 1. A wavelength tunable laser is used as the light source, and the wavenumber ( $2\pi$  times the reciprocal of the wavelength) can be scanned at any speed. When the sample is opaque, the interference signal is a simple sinusoidal wave, and its frequency is proportional to the distance, as shown in Fig. 2. Absolute distance estimation from this sinusoidal signal is the objective of this study.

The interference signal can be expressed as follows,

$$I = I_S + I_R - 2\sqrt{I_S I_R} \cos(2kL) \tag{1}$$

where  $I_{S}$ ,  $I_{R}$  are the amount of light reflected from each interface in the Fizeau interferometer, the distance between the R-S planes is L, and the wavenumber is k.

Next, consider wavenumber scanning. If the initial wavenumber is  $k_0$  and the wavenumber scanning speed is c, then the wavenumber at time t is

$$k(t) = k_0 + ct \tag{2}$$

At this time, the phase  $\phi$  at distance L is,

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$$\phi = 2(k_0 + ct)L = \phi_0 + 2\pi ft$$
(3)

where  $\phi_0$  is the initial phase, f is the frequency, and

$$\phi_0 = 2k_0 L \tag{4}$$

$$f = cL/\pi \tag{5}$$

Rewriting Eq. (1) and changing the variable definition, we get

$$I(t) = a\{1 - b\cos(2\pi f t + \phi_0)\}$$
 (6)



Fig. 1 Fizeau interference optics



Fig. 2 Interference waveform and distance

This is called a four-variable model.

Substituting Eq. (4) and (5) into this, we get,

$$I(t) = a\{1 - b\cos(2cLt + 2k_0L)\}$$
(7)

This is called a three-variable model.

# 2.2 Absolute measurement by model fitting method

The three unknowns (a, b, L) of the three-variable model in Eq. (7) are fitted by least-squares fitting with the measured values,

$$SSE(a, b, L) = \sum_{i=1}^{n} (I_i - I(t_i))^2$$
(8)

However, since the unknown variable *L* is contained in the cosine function, which is a periodic function, the fit is multimodal, and the convergence value varies with the initial value. Since the intensity value is a periodic function of approximately  $\lambda/2$ , the evaluation function *SSE* also has the same periodicity. This is shown in the simulation results (Fig. 3), where local minima exist at intervals of 300 nm. Therefore, to obtain the correct result, we need to set the initial value with an accuracy of  $\pm \lambda/4$  ( $\pm 150$  nm for  $\lambda = 600$  nm).

Furthermore, looking at the dependence on the initial value in the vicinity of the true value, when there is no noise at least, the fitting error *SSE* is minimized at the true value, as shown in Fig. 4. Therefore, the true value can be obtained by fitting with multiple initial values (multi-start method). The problem is how to narrow down the search range.

# 2.3 Rough estimation by four-variable model fitting

We consider a rough estimation method for the variable *L* that we want to find. As shown in Eq. (5), *L* and *f* are proportional. Therefore, we consider the estimation of the frequency. The required accuracy is 1/10,000, assuming an estimation accuracy of 1µm for *L* = 10 mm. The Fourier transform method, which is often used for frequency estimation, is useless. Therefore, we consider a model fitting method that uses the four-variable model in Eq. (6). That is, the least-squares fit between the observed values  $I_i$  (*i* = 1, 2, ..., *n*) and the model to find the four unknowns (*a*, *b*, *f*,  $\varphi_0$ );

$$SSE(a, b, f, \varphi_0) = \sum_{i=1}^{n} (I_i - I(t_i))^2$$
(9)

This four-variable fit has the feature that there are few locally optimal solutions and a stable and correct solution can be obtained for a wide range of initial values. The simulation results are shown in Fig. 5. For an initial value range of  $\pm 10 \,\mu\text{m}$  centered on the true value of 10 mm, the *SSE* is minimized near the true value, and accurate distance estimation can be achieved stably.

By the way, in this estimation method, accuracy and stability are in conflict. In the experiment shown in Fig. 5, with 512 data points used, an estimation error of less than 1µm was obtained for an initial value range of  $\pm 10$ µm. However, stable convergence was not achieved when the initial value error exceeded  $\pm 0.5$  mm. This is because the difference between the observed waveform and the initial model waveform is too large. Therefore, we reduce the number of data to improve the stability. Fig. 6 shows the simulation results for the case with 64 data points. For a wide range of initial values of  $\pm 1$  mm centered on the true value of 10 mm, the *SSE* is minimized near the true value, and an estimation error of less than 1µm is stably obtained.



Fig. 3 Initial value dependence of estimation error and SSE in three-variable model fitting



Fig. 4 Initial value dependence of fit results in threevariable model fitting



Fig. 5 Narrow area initial value dependence of fourvariable model fitting (number of data = 512)



Fig. 6 Wide area initial value dependence of fourvariable model fitting (number of data = 64)

### 2.4 Proposed algorithm: Three-step estimation method

We propose the following three-step estimation method. [Step 1]

A four-variable model fitting with 64 data is used to estimate the distance *L* with an estimation error of less than a few  $\mu$ m. The initial values of the variables *a* and *b* are obtained from the mean values and ranges of the intensity data. The initial value of the frequency *f* is calculated from the initial value of *L*. The initial value of phase  $\varphi_0$  is given so that the model waveform approximates the measured waveform. Calculate L from the obtained frequency f using Eq. (5).

### [Step 2]

Estimate the distance L with an estimation error of less than a few hundred nm using a four-variable model fitting with 512 data points. Use the result of the first step as the initial value. Using the obtained frequency and phase, calculate the distance L from the following equation,

$$L = \left(N + \frac{\varphi_0}{2\pi}\right) \left(\frac{\lambda_0}{2}\right) \tag{10}$$

where N is the interference order, and is expressed by the following equation,

$$N = INT \begin{bmatrix} L_f / \lambda_0 / 2 \end{pmatrix} - \frac{\varphi_0}{2\pi}$$
 (11)

where INT[] is a rounded integer value and  $L_f$  is the distance converted from the frequency, expressed as a variant of Eq. (5),

$$L_f = (\pi/c)f \tag{12}$$

[Step 3]

A three-variable model fitting is used for fine estimation. The initial values of the unknowns L are obtained from the results of the second step. This initial value is added or subtracted in steps of  $\lambda$  /2 to obtain the solution that minimizes the *SSE* (the globally optimal solution) using the multi-start method. Usually, the optimal solution is obtained by three fittings, one before and one after the starting point.

# 3. Computer experiments (1): Point measurement

### 3.1 Experimental method

- Software used: Excel
- Optimization: Solver (built in Excel)
- Intensity data: theoretically calculated values (Fig. 8)
- Average intensity value *a*: 100
- Amplitude b: 1
- Distance L: around 10 mm
- Wavenumber scanning start point  $k_0$ : 2  $\pi/600$  (Fig. 7)
- Wavenumber scanning speed c: -8.71E-7 (corresponding to a wavelength scanning speed of 0.05 nm/s)
- Imaging cycle: 30 fps
- Number of images taken: 512
- Noise: Uniform noise with amplitude  $\pm 1$  due to random numbers

## 3.2 Experimental results

#### [Step 1]

A four-variable fitting with 64 data points was performed with an initial value of 9 mm. The results are shown in Fig. 9. The frequency estimate f = -2.77379 was obtained. From this, a rough estimate of the distance L of 9,999,862 nm was obtained.





Fig. 8 Intensity waveform for computer experiment (noise amplitude = 1)



[Step 2]

Using the results obtained in the previous step, a four-variable fitting was performed. The result is shown in Fig. 10, where L = 10,000,000 nm was obtained.



[Step 3]

Using the results obtained in the previous step, a three-variable fitting of the multi-start method was performed. As shown in Table 1, L = 10,000,000 nm was correctly obtained.

Table 1 Estimation results of the third step

Trial	Initial	Solution	Error	SSE
1	10,000,000	10,000,000	0	165
2	10,000,300	10,000,300	300	188
3	9,999,700	9,999,700	-300	176

## 4. Computer experiments (2): Line measurement

### 4.1 Experimental method

A theoretical interferogram was created given 1D distance data with a step height of 1  $\mu$ m as shown in Fig. 11. The number of pixels in the horizontal direction (*x*-coordinate) was set to 164. The other conditions are the same as in the previous section.

#### **4.2 Experimental results**

The results are shown in Fig. 12. Correct estimates were obtained for the entire area, including the step.

# 5. Considerations

### (1) Effect of noise

The simulation results for the case where the noise amplitude is increased to 2 are shown in Fig. 13. Spike-like errors of  $\pm 300$  nm are scattered. A median filter may be used to remove the noise.

### (2) Improvement of calculation speed

This is an issue for implementation.

#### (3) Comparison with other methods

We estimate that our method is superior in terms of accuracy, robustness, and computation time.

# 6. Summary

We have proposed an ultra-high-precision surface profiling method for opaque objects using a Fizeau interferometer and wavelength scanning interferometry. The distance to the surface is measured absolutely by fitting a least-squares model function of the interference signal to the intensity waveform. The local solution problem in the fitting is solved by using a three-step estimation method, which enables us to measure the surface profile including large steps of the order of mm in nanometer scale. The validity of the proposed method is confirmed by computer experiments.



Fig. 11 Profile for line measurement







Fig. 13 Line measurement result (noise amplitude = 2)

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This is an English translation of the paper: Katsuichi Kitagawa and Jun Mizojiri: "High Precision Absolute Distance Measurement of Surface Topography using Wavelength Scanning Interferometry ", Proc. of Symposium on Optical Measurement and Imaging Technology (SOMIT), 5-8 (Tokyo, Oct. 2021). (original in Japanese; translated by the first author)